Quantum Indeterminacy

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Abstract

This paper explores quantum indeterminacy, as it is operative in the failure of value-definiteness for quantum observables. It first addresses questions about its existence, its nature, and its relations to extant quantum interpretations. Then, it provides a critical discussions of the main accounts of quantum indeterminacy.

1 Introduction

The history of quantum indeterminacy (QI) is as old as quantum mechanics (QM) itself.[1] One just needs to be reminded that a section in Schrödinger's (1935) paper is entitled "Are the variables really fuzzy?"[2]
Given the quantum formalism, it soon appeared legitimate to ask

> [W]hether we should think of the ψ-function as specifying a fuzzy reality for the quantum observables (Fine, 1996: 64).

If this suggestion were to be borne out, QI could represent one of the best examples of metaphysical indeterminacy (MI). The core idea behind MI is straightforward: MI affects *the world*, as opposed to our language (semantic indeterminacy), or knowledge (epistemic indeterminacy). Until recently, the very idea of MI was considered, to echo Lewis' (1986: 212), simply *unintelligible*. Such a skeptical attitude was justified both by Evans' (1978) argument against indeterminate identity, and by the success of semantic explanations of indeterminate phenomena—Fine (1975), *inter alia*.

QM provides two distinct motivations for believing that MI should be taken seriously. The first is *identity* indeterminacy, and roughly concerns the alleged lack of identity conditions for quantum particles—Lowe (1994), French & Krause (2006). The second is operative in the failure of value-definiteness for quantum observables—Darby (2010), Bokulich (2014). Accordingly, we shall call it *observables* indeterminacy. In this paper we will only cover the latter. This is not only important in and on itself. Some have claimed that it delivers insights into the nature of some quantum phenomena such as *superposition*, *entanglement*, and *quantum interference*—Calosi & Wilson (2018)—while others have claimed that it holds the key for a realist understanding of different interpretations of QM such as the Copenhagen interpretation—Bokulich (2014)—and the relational interpretation—Calosi & Mariani (2020).

2 Observable Indeterminacy

QM seems to violate the classical supposition of 'value definiteness', according to which the observables of a physical system have precise values at all times. This is best appreciated

[1] We assume some familiarity with elementary QM and its interpretations.

[2] See Schrödinger (1935). This is the paper in which Schrödinger introduces his infamous cat thought-experiment. To be clear, Schrödinger answers negatively.

in the presence of the **Eigenstate-Eigenvalue Link**:[3]

Eigenstate-Eigenvalue Link (EEL). A physical system s has a definite value v of an observable $\mathcal{O}$ *iff* the state of s is an eigenstate of $\mathcal{O}$ that belongs to v.

Calosi & Wilson (2018) provides a threefold classification of cases supporting the existence of observable QI:

Superposition. A linear combination $|\omega\rangle = c_1|\psi\rangle + c_2|\phi\rangle$ of different eigenstates $|\psi\rangle$ and $|\phi\rangle$ of an observable $\mathcal{O}$ is not always an eigenstate of $\mathcal{O}$. If a system S is in $|\omega\rangle$ it does not have a definite value of $\mathcal{O}$.

Incompatible Observables. Consider two observables $\mathcal{O}_1$ and $\mathcal{O}_2$. The observables commute *iff* $[\mathcal{O}_1, \mathcal{O}_2] = \mathcal{O}_1\mathcal{O}_2 - \mathcal{O}_2\mathcal{O}_1 = 0$. If they do not, they are *incompatible*. If two observables are incompatible, they do not share all the same eigenstates. Thus if S is in one such non-shared eigenstate of $\mathcal{O}_1$ ($\mathcal{O}_2$), it follows that it does not have a definite value for $\mathcal{O}_2$ ($\mathcal{O}_1$).

Entanglement. Consider an *entangled* system S_{12} composed by S_1 and S_2 with corresponding Hilbert space $\mathcal{H}_{12} = \mathcal{H}_1 \otimes \mathcal{H}_2$. S_{12} might be in an eigenstate $|\omega\rangle$ of $\mathcal{O}_{12} = \mathcal{O}_1 - \mathcal{O}_2$ that is neither an eigenstate of $\mathcal{O}_1$ nor an eigenstate of $\mathcal{O}_2$—with $\mathcal{O}_1$ and $\mathcal{O}_2$ defined on $\mathcal{H}_1$ and on $\mathcal{H}_2$ respectively. Both S_1 and S_2 will therefore lack a definite value for the corresponding observables.

The general suggestion is that, in all the cases above, the QI resulting from the failure of value-definiteness represents a genuine case of MI, in that the value of the relevant observables is indeed *not determinate*. This indeterminacy, at first sight, is not due to the language we use to describe the quantum phenomena, nor is it due to our ignorance about the physical details of a certain system. As we are about to see, things are not that straightforward, as there are ways to resist the claim that there is any QI after all.

3 Against Observable Indeterminacy

The suggestion that there is *observable* indeterminacy has been recently challenged. Glick (2017) provides two arguments against it.

3.1 The Fundamentality Argument

The first argument is as follows:

No Fundamental QI According to the main realist interpretations of QM—Bohmian mechanics, spontaneous collapse theories, and Everettian QM—there is no fundamental QI (205).

Eliminability. Derivative QI is eliminable (206).[4]

Conclusion. There is no QI.

Glick backs up **No Fundamental QI** for each main realist interpretation:

> [F]irst, and most straightforwardly, the Bohm theory endows particles with determinate positions and momenta at all times [...] Second, the Everett interpretation, as developed by Wallace (2012), recognizes only the universal wavefunction in its fundamental ontology. The universal wavefunction is perfectly determinate at every time [...] Finally, consider dynamical collapse theories

[3] As Lewis (2016) notes this is the somewhat standard way of reading off the physical properties out of the quantum formalism. See Wallace (2016) and Gilton (2016) for a critical discussion. Lewis (2016) argues that there is QI under many other weaker links.

[4] We take "derivative" and "non-fundamental" as synonymous.

> such as versions of GRW. The two versions of the GRW adopted by most contemporary defenders are the mass-density and flash-ontology varieties. Neither contains fundamental indeterminacy: the distribution of mass-density and the location of the flashes are both perfectly determinate. (205)

Having argued for **No Fundamental QI**, Glick moves on to **Eliminability**:

> [A]ny indeterminacy would occur at the non-fundamental level and *hence may be viewed as eliminable*. (206, italics added)

Glick does not provide direct support for **Eliminability**, and simply suggests that derivative QI *may be viewed* as eliminable. As it stands, **Eliminability** seems ambiguous between the following two readings:

Strong Eliminability. Derivative QI is eliminable *tout-court*.

Weak Eliminability. Derivative QI is eliminable *qua metaphysical*.

On the former reading, there simply is no QI at all. On the latter, weaker reading, QI still exists, but the fact that it is derivative implies that it is not metaphysical. Both readings support Glick's conclusion if we are to consider QI as an example of MI. We urge to keep the two readings distinct insofar as potential responses to them might differ significantly.

3.2 The Sparse View Argument

The first premise of the *Fundamentality Argument* establishes that there is no QI according to the main realist interpretations of QM. Glick concedes that one could—and indeed should—look elsewhere for fundamental QI, namely to the Copenhagen interpretation. However, Glick argues, nothing in the Copenhagen interpretation forces us to accept QI either. This is because a different understanding of the same interpretation dispenses with QI altogether. Glick calls this understanding the **Sparse View**:[5]

Sparse View. When the quantum state of a physical system S is not in an eigenstate of an observable $\mathcal{O}$, S *lacks both* the determinable and the determinate associated with $\mathcal{O}$.

To illustrate, consider a system S is a superposition of spin_x states. According to the **Sparse View**, S not only does not have any definite value of spin_x, it does not have spin_x either. The sheer availability of the **Sparse View** shows that the Copenhagen interpretation of QM does not support the existence of QI.

3.3 Responses

Let us start by considering possible responses to the *Fundamentality Argument*. Both **No Fundamental QI** and **Eliminativism** can be resisted. Here we shall focus on the latter, whereas the next section will be dedicated to the former.

Recall that **Eliminativism** has a strong and a weak reading. Against **Strong Eliminativism** we point out that the derivativeness of a given entity hardly entails its eliminability. In general, eliminativism about derivative phenomena is highly revisionary.[6] Consider three notions that are usually taken to track the relation between fundamental and derivative levels: *emergence*, *grounding*, and *reduction*. The first two are generally introduced precisely *in contrast* to eliminativism. As regards to *reduction*, two distinct varieties have been discussed in the literature, a *conservative* and an *eliminative* one.[7]

[5] For determinables and determinates see §5.

[6] Variants of this argument are in Calosi & Mariani (2020), and Calosi & Wilson (MS).

[7] We set aside *inter-theoretic reduction*, since the quantum state and the observables belong to the same theory.

Only the latter supports the *Fundamentality Argument*. However, *eliminative reduction* is generally motivated by the claim that the derivative entity is *replaced* by a new entity, as for example with oxygen replacing phlogiston—van Riel & Van Gulick (2003). Since nothing replaces quantum observables, the conclusion is that **Strong Eliminativism** seems unconvincing.

According to **Weak Eliminativism** QI is not metaphysical after all. This suggests that we can account for it in broadly representational terms. In response, we should register that, at least *prima facie*, QI looks very different from the phenomena that have been the target of the main representationalist accounts of indeterminacy. For one, QI is not *Sorite's paradox* susceptible. Nor does it have to do with compositional vagueness, or the problem of the many. Furthermore, representationalist approaches of the semantic variety assume that the predicates of the relevant languages are *vague*. By contrast, the mathematical language of QM does not seem to be semantically defective. Traditional representational approaches of the epistemic variety, when applied to the case at hand, would entail that the quantum world is maximally precise, and we are just ignorant either about what precise way the quantum world really is, or about what is the correct *use* of the quantum language—Williamson (1994). But this seems to run afoul of several foundational result in QM, such as the Gleason's (1957) and the Kochen-Specker's (1967) theorems.[8]

The considerations above are not meant to show that a representationalist treatment of QI is *in principle impossible*. We just want to point out that much more needs to be done in order to show that either (i) already existing representationalists accounts work for QI, or (ii) that a new, so far undeveloped representationalist treatment of QI is viable.

Let us now turn to to the *Sparse View Argument*. Here a possible response is that, all things considered, the **Sparse View** is explanatory inferior to the somewhat usual take on standard QM, for it does not fare well with respect to particular observables. A case in point would be *position*. It follows from the **Sparse View** that a system S with no definite position has no determinable position property either. In particular it does not have the *maximally unspecific determinable* position.[9] But, arguably, the maximally unspecific determinable position is just "being in space". It would follow that every system in an eingenstate of momentum is not in space, which seems quite discomforting.

4 QI and Interpretations of QM

The **No Fundamental QI** premise of Glick's *Fundamentality Argument* is the claim that according to the major realist interpretations of QM, QI is at best derivative. This raises the independently interesting question about the existence and nature of QI within particular interpretations of QM.

We introduced *observable* indeterminacy via the EEL. As a matter of fact, the large part of recent contributions on QI has explicitly focused on the Copenhagen interpretation, the one that features the EEL prominently. Bokulich (2014), for instance, expresses the hope that a better understanding of QI could justify a *realist* approach to this interpretation, which is usually considered anti-realist:

> [O]ne might object that the standard interpretation of quantum mechanics is only an instrumentalist theory [...] I think this objection is a mistake: For any theory one can take either a realist or instrumentalist attitude towards it [...] In this paper I am taking a realist attitude towards the standard interpretation, and asking what the world would be like if this interpretation were true (460).

[8] We will return to this in §5.

[9] Roughly, determinable property that is not a determinate of any other determinable. One possible reply is to contend this is in fact *not* the determinable that is being rejected here, perhaps because "having a precise position" and "having position" are not determinable-determinate related. Thanks to David Glick here. Another suggestion has it that S might have "counterfactual" properties expressed by the predicates "having a (precise) position if measured". We owe this suggestion to Peter Lewis.

Similarly, Skow (2010) is explicit that *observables* indeterminacy affects the Copenhagen interpretation, and contends it is useful in providing a satisfactory (realist) understanding of the theory.

When we pass to the major realist interpretations of the theory, we should register that the EEL is either rejected or significantly revised. It is thus a substantive question whether *any* form of indeterminacy is retained in those interpretations.

As Glick rightly notices, in Bohmian mechanics position and momentum have always definite values. Does this mean that *all* properties have definite values, and therefore there is no observable indeterminacy? According to Lewis this is not the case. Consider spin. And suppose a Bohmian particle in region r is in the following state: $\frac{1}{\sqrt{2}}(|r,\uparrow\rangle + |r,\downarrow\rangle)$. According to Lewis this Bohmian particle has *indeterminate spin*, insofar as its position does not pick out one spin value over the other. If we perform a spin measurement, we correlate spin with position and end up with, say, $\frac{1}{\sqrt{2}}(|r_1,\uparrow\rangle + |r_2,\downarrow\rangle)$, where r_1 and r_2 are disjoint subregions of r. After such a measurement the position of the Bohmian particle does pick out a definite spin value, yet:

> [T]he result of the measurement reflects a determinate spin property that the electron has after the measurement, *but not a determinate spin property* that the electron had prior to measurement (Lewis, 2016: 102, italics added).

One can take issue with Lewis's argument. We simply note here that the argument turns on the fact that spin is a physically relevant property. Contemporary defenders of Bohmian mechanics will not endorse such an assumption. As a matter of fact, many contemporary Bohmians subscribe to the view that the only physically significant properties are a (proper) subset of the *definite valued properties* of Bohmian particles—see e.g. Miller (2013) and Esfled et al. (2014). In this case, there would be neither fundamental, nor derivative observable indeterminacy in Bohmian mechanics.

Let us now consider Everettian QM (EQM; Everett, 1957). As it is well known EQM comes in different varieties, only some of which feature QI. For example, Deustch (1985: 20) suggests that for each term in the universal quantum state there is an infinite number of worlds measured by the Born-rule. Arguably, if one adds continuously many infinite-measure sets of worlds to the ontology of EQM, there would not be any QI. Each of these worlds will be completely determined. Many contemporary defenders of EQM would however subscribe to the so-called "Decoherence Only" variant of the theory, in which the universal quantum state is the only fundamental object and different Everettian worlds are *derivative*, *emergent* entities represented by individual decoherent histories—Wallace (2012). Decoherent histories are represented by time-ordered sequences of time dependent orthogonal projection operators—summing up to unity—that stand for observables of the *whole state of the world at a time*—Wilson (2020: 83). These histories are required to be sufficiently well-decohered so as to be dynamically independent. One form of indeterminacy results from the fine-grainedness of the chosen partition into different histories. Alastair Wilson, in his (2020), calls it indeterminacy in *world number*.[10] This is radically different from the indeterminacy we consider here, so we will leave it at that. Note however that the projection operators representing different decoherent histories stand for observables of the whole world. What about subsystems within such worlds? Can they display the kind of observable indeterminacy that we focused on here? According to Wilson the answer is yes:

[10] It is unclear whether this indeterminacy is metaphysical in nature. One can push the point that the predicate "being a world" is just *semantically vague*. On top of it, it seems that this sort of vagueness is *familiar*, in that it seems the vagueness associated with higher-order terms in science. If so, there would be nothing QM-specific about it. Thanks to an anonymous referee here. It is also substantive question whether Wallace's *patternism*—roughly the view that physical systems are patterns in the wavefunction—is a first step towards providing a weak-eliminativist story, insofar as one con recognize that the predicate *pattern* is *semantically vague*. Thanks to Peter Lewis here.

> Everett worlds are themselves indeterminate with respect to the microscopic physical quantities that they instantiate (Wilson, 2020: 180).

This is because in any Everettian world there might be physical subsystem whose wavefunction is not well-decohered. In that case, the Everettian world will fail to settle which determinate properties the physical subsystem in question instantiates. In effect, we will have cases of QI *every time* the wavefunction of the relevant physical system does not decohere and interference effects cease to be negligible. Given that decoherence is much more rapid for systems with many degrees of freedom, QI would be mostly confined to somewhat microscopic states of affairs and subsystems. One important example is the double-slit experiment, in which interference effects are clearly non-negligible. It represents a case of *observable* indeterminacy with respect to position. Wilson is explicit:

> Second, Everett worlds are indeterminate in *nature*: a world for example may fail to determine which of the two slits an electron travels through, if the electron wavefunction does not decohere in the process (Wilson, 2020: 172).

Lewis agrees:

> So, in the many worlds theory, too, microscopic systems can have indeterminate properties, where this indeterminacy is primitive, and has nothing to do with (...) familiar kinds of vagueness (Lewis, 2016: 97).

Note that Glick might agree with all this. In the end, this QI is explicitly *derivative*. Yet, it is unclear that once this variety of QI is explicitly recognized in EQM, it is in fact *eliminable*. This seems the be the case for **Strong Eliminability**. Strong-elimination of QI would result either in the elimination of Everettian worlds themselves—for they are the ones that are indeterminate—or in adding structure to the universal quantum state, as per Deustch (1985) proposal. Both options seem to run contrary the very spirit of Decoherence Only EQM. **Weak Eliminability** is surely preferable here, but no broadly representational account of QI in EQM has been proposed in the literature. As Wilson concludes:[11]

> [I]ndeterminacy in world nature may be thought of as a naturalistic form of *metaphysical indeterminacy* (Wilson, 2020: 182).

Let us finally consider *spontaneous collapse interpretations*, of which GRW—Ghirardi, Rimini & Weber (1986)—is the most prominent example. In recent years there has been a lively debate about the correct ontology of GRW. The main divide concerns whether GRW is ultimately about the high-dimensional wavefunction of the universe ψ—a view sometimes called $\mathrm{GRW}_\emptyset$, defended among others by Albert (1996)—or whether the theory needs to posit a so-called *Primitive Ontology* (PO) in 3D-space (Allori *et al*, 2008).

[11] For the sake of completeness we should note that there are other versions of EQM that are even more favourable to MI. One such version is what Barrett calls the *Bare Theory.* As Barrett puts it:

> [T]he bare theory is simply the standard von Neumann-Dirac formulation of QM with the standard interpretation of states (the eigenvalue-eigenstate link), but stripped of the collapse postulate—hence *bare* (Barrett, 1999: 94).

Given the endorsement of the EEL it's not difficult to see that superposition states will give rise to indeterminacy. Wallace is explicit:

> [H]ence, given the eigenvalue-eigenstate link the cat is in an *indefinite* state of aliveness (Wallace, 2008: 42), italics added.

The same goes for the so-called *Many Minds* theory. Once again, Wallace recognizes it:

> [A] state like (2.49) [a superposition state] *really is indefinite* (Wallace, 2008: 40, italics added).

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The PO approach comes in two major varieties. According to the Mass Density Model (GRW_M; Ghirardi Grassi & Benatti, 1995), the ontology is given by the universal mass-field. According to the Flash Model (GRW_F; Tumulka, 2006), the ontology is constituted by events in 3D-space individuated by the collapse of ψ. The status of QI within $GRW_\emptyset$ and GRW_F does not reveal anything novel: QI is not fundamental and yet it is a substantive issue whether its being derivative entails any form of eliminativism. The status of QI within GRW_M, on the other hand, is particularly interesting. Glick claims that "the distribution of mass-density [is] perfectly determinate" (2017: 205). It could even be argued that the introduction of the mass-density in GRW was motivated, at least partially, by the hope that any indeterminacy would disappear. Lewis (2016: 87) is quite clear that GRW with the standard EEL would entail a widespread, radical indeterminacy because of the so-called *tails problem*. Very roughly, the dynamical evolution of GRW never evolves into eigenstates, the wavefunction having *tails* stretching to infinity. The introduction of the mass-density as a PO should overcome this problem, exactly because the mass distribution is supposed to be always determinate. However, on a closer inspection this is not so obvious. The fundamental ontology of GRW_M is given by the Mass Density function $\mathcal{M}$, which translates the properties of the Hilbert space onto the distribution of mass in ordinary 3D-space, therefore acting as a substitute to the EEL. But, as first noticed by Ghirardi Grassi & Benatti (1995), and later discussed by Clifton & Monton (2000), only a portion of the solutions to $\mathcal{M}$ are indeed determinate, while some others are not.

It is instructive to look at the following example in Bassi & Ghirardi (2004). Consider a system of N particles and two congruent regions r_1 and r_2. Next, consider two states: (i) $|\psi^{\oplus}\rangle = \frac{1}{\sqrt{2}}[|\psi_N^{r_1}\rangle + |\psi_N^{r_2}\rangle]$ and (ii) $|\psi^{\otimes}\rangle = |\phi_{N/2}^{r_1}\rangle \otimes |\phi_{N/2}^{r_2}\rangle$. As shown in detail in Clifton & Monton (2000), (i) and (ii) give rise to *the same mass density function* $\mathcal{M}$. And yet the situations they describe are physically different: in the tensor product case (ii), the mass density is *determinately spread* across the two regions evenly. By contrast, in the superposition state (i), the mass density is *neither determinately* in r_1, *nor determinately* in r_2. The question becomes whether we could exclude states like (i) from the domain of $\mathcal{M}$. For instance, Bassi & Ghirardi (2004) define a relation of accessibility for mass in order to restrict the domain of $\mathcal{M}$, and conclude that states such as (i) are correctly excluded. Tumulka (2011: 8-9) disagrees, arguing that such a view entails that the ontology given by $\mathcal{M}$ would depend on what observers can experimentally access. If Tumulka's objection is correct, superpositions like (i) could be considered states for which $\mathcal{M}$ generates *indeterminacy in the mass distribution*—the argument is presented in detail in Mariani (2020). See also McQueen (2015). It is not implausible to read the following passage as suggesting something similar:

> [T]hat is, if the square amplitude of the wavefunction assigns a weight of .25 to a configuration in which a particular electron is on the left and a weight of .75 to a configuration in which the electron is on the right, then *somehow .25 of the matter of the electron is on the left, and the other .75 is on the right.* Since each possible configuration assigns an exact position to each particle, the weighting of the configurations can in this way be used to define a matter distribution for each particle. *The matter of the particle literally gets smeared out over space* (Maudlin, 2019: 117, italics added).

In light of the above, it seems safe to conclude that it is far from clear that GRW_M is free of fundamental indeterminacy.

To conclude this section we should briefly mention another way of rejecting Glick's **No Fundamental QI**. This consists in resisting the very idea that we should limit ourselves to the three interpretations above. There are other interpretations of QM that are somewhat hospitable to fundamental QI. For instance, Calosi & Mariani (2020) considers the Relational Interpretation of QM (RQM)—Rovelli (1996)—and contends that it provides examples of fundamental QI. Similar considerations seem to apply also to the Modal Hamiltonian Interpretation—Lombardi (2019). If this is correct, it seems that Glick's restriction is unjustified. After all, it would be ad-hoc to restrict one's look to those interpretations in which QI is derivative.[12]

5 Accounts of Quantum Indeterminacy

If one finds the arguments in §3-4 compelling, one is left with QI. The question becomes: how are we to account for it? In general, we can divide accounts of MI into *meta-level* and *object-level* accounts—Wilson (2013). As Wilson (2013) notes, the most general and profound difference between these accounts is that, according to the former *it is indeterminate which determinate state of affairs obtains* (SOA), whereas according to the latter *it is determinate that an indeterminate SOA obtains*. This difference is crucial to assess the viability of different approaches to QI.[13]

5.1 Metaphysical Supervaluationism

Metaphysical Supervaluationism (MS) is arguably the leading meta-level account of MI. The idea behind MS—Barnes & Williams (2011)—is that MI is indeterminacy in which SOA of a range of admissible precisifications, i.e precise, determinate SOA,[14] obtains—or correctly represents the actual world. As Barnes (2010) writes:

> It's perfectly determinate that everything is precise, but [...] it's indeterminate which precise way things are (622).

The admissible precisifications—particular SOA, or more in general, worlds—are those that *do not determinately misrepresent reality*.[15] In the light of the above, we can capture the core of MS as follows:

Metaphysical Supervaluationism. It is metaphysically indeterminate whether P *iff* there are two possibly admissible, exhaustive and exclusive SOA, the SOA that p and the SOA that $\neg p$, and it is *indeterminate* which one obtains.[16]

The application of MS to QI seems to be straightforward enough: just identify the terms of a superposition state with different precisifications. As Darby (2010) writes:

> [There is] a suggestive parallel between the terms in the superposition and the idea [...] of precisifications. One of the terms in the superposition [...] is a term where the cat is alive, the other is not; that is reminiscent of multiple ways of

[12] One possible response is as follows. Glick only considers *realist* interpretations of QM. And realist interpretations of QM take a realist stance towards the quantum state. By contrast, Rovelli is explicitly anti-realist about it—Rovelli (2018). However one can presumably be realist about QM *without thereby being realist about the quantum state*. For example, one can be realist about quantum observables—and this is enough in the present context. As a matter of fact, as Calosi & Mariani (2020) notes, Rovelli is explicitly realist about quantum observables, quantum systems and quantum interactions.

[13] One upshot of the previous section is that there is a reasonable case to be made that there is indeterminacy in some (realist) interpretations of QM that do not feature the EEL. By contrast, one may notice, the proponents of the accounts we are about to explore usually "use" the EEL to argue that there is QI in the first place. These last two claims—so the thought goes—are in *tension*. It should be noted however that none of the accounts we are about to explore *depend* on the EEL. In effect the accounts—at least in some respects—are supposed to work for metaphysical indeterminacy in general, not just for QI. Thanks to an anonymous referee here.

[14] The notion of *precisification* is modeled on the supervaluationist theory of vagueness—Fine (1975)—on which a precisification is a complete and maximal set of sentences with a determinate truth value.

[15] This is the counterpart of the semantic requirement that admissible precisifications of a predicate are compatible with the actual use of that predicate.

[16] This formulation is found almost *verbatim* in Barnes & Williams (2011: 113-114). Note that *indeterminacy* appears in the very characterization of MS. This is because, as Barnes and Williams explicitly admit, MS offers a *non-reductive* account of MI.

> drawing the extension of 'alive', on some of which 'the cat is alive' comes out true, on some, false (235).

In general, consider a system S in state $|\omega\rangle = c_1|\psi\rangle + c_2|\phi\rangle$. There is MI because there are two admissible precisifications, the SOA that ψ and that ϕ respectively, and it is indeterminate which one is the case.[17] That is, superposition indeterminacy boils down to indeterminacy about which term of the superposition obtains.

Crucially, the precisifications that are identified with superposition terms are *maximal*—or *complete*—and *classical*. In other words, they settle everything, and are indeterminacy-free:

> Importantly, given our picture of indeterminacy, all the worlds in the space of precisifications are themselves maximal and classical (Barnes & Williams, 2011: 116).

It is exactly because of this that MS can stick to classical logic and classic compositional semantics, one of the selling points of the account.

5.2 The Determinable Based Account

The most developed object-level account of MI is the determinable-based account (DBA). First introduced in Wilson (2013), it was applied to QI in Calosi & Wilson (2018). The core idea is that MI involves the obtaining of an indeterminate SOA—roughly a SOA where a constitutive object fails to have a unique determinate of a determinable:

> **Determinable-based MI**: What it is for an SOA to be MI in a given respect R at a time t is for the SOA to constitutively involve an object (more generally, entity) O such that (i) O has a determinable property P at t, and (ii) for some level L of determination of P, O does not have a unique level-L determinate of P at t (Wilson, 2013: 366).

There are two ways in which an object can fail to have a unique determinate of a determinable: either it has *none*—*gappy* MI—or it has *more than one*—*glutty* MI. Glutty MI has been cashed out in at least two ways:[18] one where multiple determinates are instantiated, albeit in *relativized* fashion, and one where multiple determinates are instantiated, each to a *degree* less than one—where the degree is to be read off the coefficients in the quantum state. By way of illustration, consider a system S in state $|\omega\rangle$. Suppose we are to focus on an observable $\mathcal{O}$ with two eingenvectors $|\psi\rangle$ and $|\phi\rangle$ belonging to different eigenvalues v_1 and v_2, respectively. First, write the quantum state in the basis of $\mathcal{O}$'s eigenvectors. This in general will be $|\omega\rangle = c_1|\psi\rangle + c_2|\phi\rangle$. According to *gappy* MI, S has the determinable associated with $\mathcal{O}$ but no value of $\mathcal{O}$, i.e. neither $\mathcal{O} = v_1$, nor $\mathcal{O} = v_2$. According to *glutty* MI, S has both $\mathcal{O} = v_1$ and $\mathcal{O} = v_2$, either in a relativized fashion or to a degree $|c_1|^2 < 1$, $|c_2|^2 < 1$ respectively. Similar considerations apply to QI stemming from *incompatible observables* and *entanglement*.[19]

5.3 Against the Accounts

The major challenge to MS comes from some foundational no-go theorems in QM, such as the Kochen-Specker theorem—Darby (2010), Skow (2010), Calosi & Wilson (2018).[20] The

[17] For the sake of completeness we should note that this straightforward application raises questions on how to understand the coefficients c_1 and c_2 in the quantum state.

[18] See Calosi and Wilson (2018).

[19] The *incompatible observables* case is straightforward in that an eigenstate of $\mathcal{O}_1$ is a superpositon of eigenstates of the incompatible $\mathcal{O}_2$. The case of *entanglement* is a little more difficult. For a detailed account see Calosi & Wilson (2018).

[20] Torza (2020) argues along the same lines using only incompatible observables.

argument is simple. MS crucially requires all precisifications to be maximal and classical. For an admissible precisification to be both maximal and classical is for it to attribute a definite value to *all* quantum observables at any given time. And this runs afoul of the Kochen-Speker theorem.[21]

The problems with the DBA can be divided in general problems of the account, and specific problems of its *gappy* and *glutty* implementation. In general, one problem is that it is controversial whether quantum observables do in fact have a determinable-determinate structure—Wolff (2015). Furthermore, the DBA explicit commitment to the non reducibility of determinables in terms of logical construction of determinates may be found problematic—Torza (2020).[22] As for specific problems, there can be a worry that *gappy* implementations do not have the resources to account for quantum interference, and that they wash away important quantum information stored in the coefficients of a given superposition state—Calosi & Wilson (2018). *Glutty* QI faces different challenges depending on whether it comes in the relativization or degree variant. As for the former, it is sometimes unclear what the relativization target is in simple superposition cases, and whether it provides an example of MI after all. The thought is that the relevant situation can be constructed as the obtaining of *completely determinate yet relational states of affairs*—Calosi & Mariani (2020). The major challenge for *glutty* QI in its degree variant is that many find the very notion of degree-instantiation simply unintellegible—Rosen & Smith (2004). We leave possible responses to the following section.

6 Future Developments

The debate on QI is far from being settled, and a large number of new approaches are emerging. In this final section we shall mention some, and suggest possible ways for the debate to move forward, with an eye to possible responses to the main challenges for each of the three main views we have considered, *Eliminativism*, *Metaphysical Supervaluationism*, and the *Determinable-Based Account*. Starting from *Eliminativism*, a suggestion has it that it is a *desideratum*, for any physical theory, to be a precise description of its target phenomena.[23] If this is on the right track, then it seems we have *independent methodological grounds* for a strong-elimination of QI. There are prospects for weak-eliminativism too. There seem to be two different options here. First, one could argue that QI, despite first appearances, is not different from other cases of indeterminacy, and then apply existing models of representational indeterminacy such as *supervaluationism*. Second, one could concede that QI is of a special kind, and then provide a novel representational account of it.

Moving to *Metaphysical Supervaluationism*, there have been attempts to provide sophisticated variants of it, that are allegedly capable to face the threat from QI. Recall what the problem was: metaphysical supervaluationism requires precisifications to be both *precise* and *maximal*, or *complete*. Foundational quantum results rule such precisifications out. Let us simplify slightly, and use *incompatible observable* indeterminacy to make our point. Suppose a quantum system is in state: $|\uparrow_x\rangle = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle + |\downarrow_z\rangle)$. Then, according to supervaluationism there are two *complete* precisifications $P_1 = \{|\uparrow_x\rangle, |\uparrow_z\rangle$, and $P_2 = \{|\uparrow_x\rangle, |\downarrow_z\rangle$, and it is unsettled which one obtains. The problem is that P_1 and P_2 cannot represent quantum worlds, insofar as $spin_x$ and $spin_z$ are incompatible observables. As both Torza (2020) and Darby and Pickup (2019) notice, the problem is that supervau-

[21] More precisely, the theorem states that in a Hilbert space with dimension $d > 3$, it is *impossible* to assign a definite value 0 or 1 to every projection operator $\mathcal{P}_i$ such that, if a set of commuting $\mathcal{P}_i$ satisfies $\sum P_i = 1$, then the values $v(P_i)$ associated with such projectors satisfies $\sum v(P_i) = 1$.

[22] In particular it entails that a widely held reductionist view of determinables, disjunctivism—roughly the view that determinables are disjuncts of determinates—is untenable. Note that this affects only *gappy* MI.

[23] One can even distinguish between *indeterminacy* and *imprecision*, and require that *determinacy* alone is a *desideratum*. Thanks to David Glick here.

lationism requires precisification to be *complete*, that is to settle *everything* about a given world. Thus, they suggest, we should simply reject the *completeness* requirement and allow precisifications to be *partial*. This common idea is fleshed out in different ways. According to Torza,[24] in the case above there is indeed only one *partial precisification* $P = |\uparrow_x\rangle$. P contains neither $|\uparrow_z\rangle$, nor $|\downarrow_z\rangle$. As a result, the proposition "The system has a definite $spin_z$" is neither true nor false, and, as Torza writes:

> [R]eality is metaphysically indeterminate when some sentence formulated in a semantically nondefective language is neither true nor false (Torza, 2020: 4257).

By contrast, Darby and Pickup advocate the use of *situation semantics* to model indeterminacy. The key insight is that situations are, by their own nature, always *partial*. If sentences are evaluated at partial situations, so the thought goes, the problems for MS vanish. In the case at hand, there will be *three* relevant situations, $S_1 = |\uparrow_z\rangle$, $S_2 = |\uparrow_z\rangle$, and S_3, which is the fusion of S_1 and S_2.[25] In S_1 it is true that the system has $spin_z = up$ and false that it has $spin_z = down$. In S_2, it is false that the system has $spin_z = up$ and true that it has $spin_z = down$. In S_3 both sentences are neither true nor false.[26]

> A proposition p is indeterminate iff it is true in some situation which is a candidate for representing reality and false in some other such situation (Darby and Pickup, 2019).

Finally let us briefly consider the *Determinable Based Account*. We saw that Wolff (2015) raises doubts that quantum observables exhibit a determinable-determinate structure, especially because they require a significant revision of such a structure.[27] But she concedes that

> [W]hether we should revise the determinables/determinates model to apply to the case of spin and other quantum properties will depend upon whether the model proves useful elsewhere as well (385).

A first step in this direction is in Calosi (Forthcoming) in which such a revision is required in order to account for locations of material objects. The basic idea is that the literature on formal theories of location provides different examples in which a given material object has the determinable location, or position, but lacks a unique determinate position—exactly as is required by the DBA. For example, point-particles in gunky spacetimes have the determinable position *without having a determinate* of it. And a particular version of an influential metaphysics of persistence, namely *locational endurantism*, is committed to *multilocation*, i.e. the claim that one object *has more than one location*.

The case for *glutty* MI in its relativization variant can be bolstered in different ways. First, it seems that in the case of *entanglement* relativization targets are indeed available. Consider any composite entangled system, and suppose you are interested in system S that is part of it. The state of the entire composite system can be written down as $|\omega\rangle = \sum_i a_i |\psi\rangle_{S_i} |\psi\rangle_{S'_i}$, where S' is the mereological complement of S. Call state $|\psi\rangle_{S'_i}$ the "state of S' relative to S being in state $|\psi\rangle_{S_i}$".[28] This can be used to give a *glutty* relational QI as follows. The different $|\psi\rangle_{S_i}$ are eigenstates of some observables $\mathcal{O}$ belonging to a particular eingenvalue v. We say that the system S has property $\mathcal{O} = v$ relative to the state of S' being its relative state as defined above. For the sake of illustration, consider the spin$_x$-state: $|\omega\rangle = \frac{1}{\sqrt{2}}(|\uparrow_x\rangle_1 |\downarrow_x\rangle_2 + |\downarrow_x\rangle_1 |\uparrow_x\rangle_2)$. We say that particle 1 has spin$_x$-up relative to particle 2 being spin$_x$-down, and particle 1 has spin$_x$-down relative to particle 2 being spin$_x$-up—ditto for particle 2. Second, one may find particular interpretations of QM in which

[24] In what follows we slightly abuse terminology and notation to keep them in line with the rest of the paper.

[25] For details about situations and their fusion we refer to the original paper.

[26] Darby and Pickup defend this very point at length, but it is not the most pressing issue here.

[27] In particular, they require to give up widely held axioms of determination such as *Requisite determination* and *Unique determination*. See Wilson (2017).

[28] For a similar characterization of Relative-State EQM see Conroy (2012).

relativization targets are readily available. Calosi & Mariani (2020) addresses at length the issue in the context of RQM, the interpretation championed by the leading physicist Carlo Rovelli. As Rovelli himself puts it, the *main* tenet of RQM is the *relativization of states and observables of physical systems to other physical systems*:

> [T]he actual value of *all* physical quantities of *any* system is only meaningful in relation to another system (Rovelli, 2018: 6).

It is then a natural solution to choose other physical systems as relativization targets for *glutty* MI.

Finally, a possible development for degree-*glutty* MI is to provide other examples in which degree-instantiation plays a role. Wavefunction realism might be a case in point as degree-instantiation is explicitly advocated in e.g. Ney (2020).

Ney suggests that wavefunction realists should recover a three-dimensional ontology by allowing the wavefunction to *instantiate particular particle configurations to a degree*:

> The suggestion is to make this a bit more precise by allowing that alternative particle configurations may be instantiated to degrees corresponding to the amplitude-squared of the wavefunction at the points corresponding to these configurations (...) Although *these particles do not have determinate locations, they may instantiate multiple locations to various degrees* (Ney, 2020: 4246, italics added).

The similarity with *glutty* MI in its degree variant is indeed striking.[29] Another fascinating, yet virtually unexplored suggestion, is to relate degrees of instantiation with objective probabilities.[30]

A long, perhaps indeterminate path seems to lie ahead of us.

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[29] We are indebted to two anonymous referees for suggestions and comments.

[30] The fact that degrees do correspond to probabilities given by the Born rule cries out for an explanation.

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